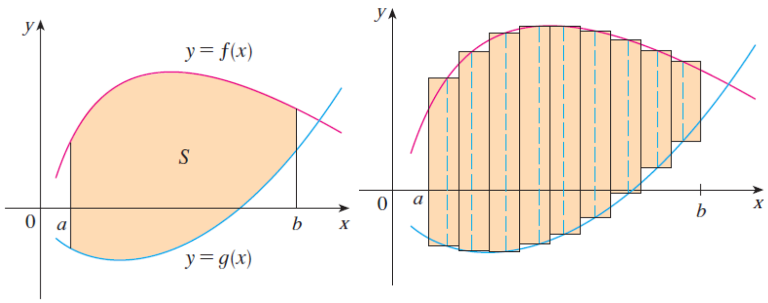
Closing Next Wed: HW_3A,3B,3C

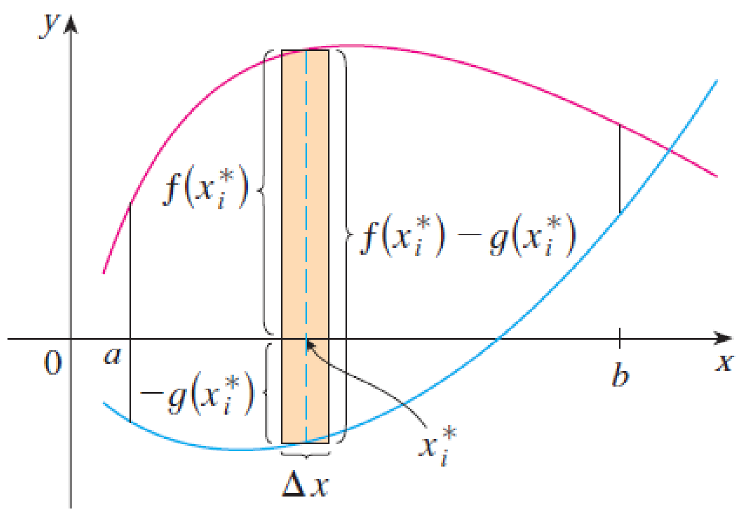
Exam 1 is next Thurs (4.9, 5.1-5.5, 6.1-6.3)

Ch 6: Basic Integral Applications

<u>6.1 Areas Between Curves</u>

Using dx:

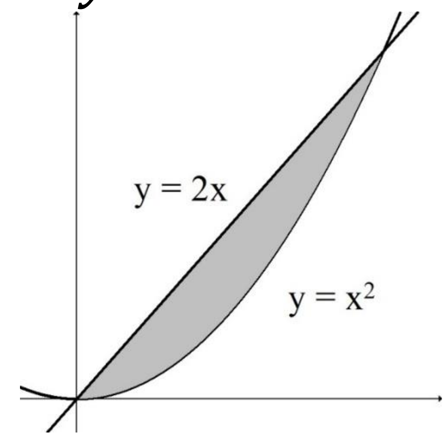




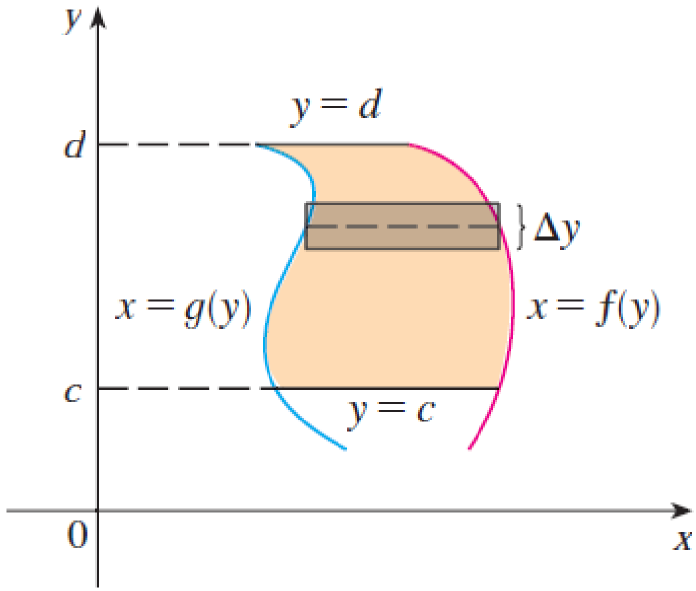
(a) Typical rectangle

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} (f(x_i) - g(x_i))\Delta x$$

Example: Find the area bounded between $y = 2x$ and $y = x^2$.

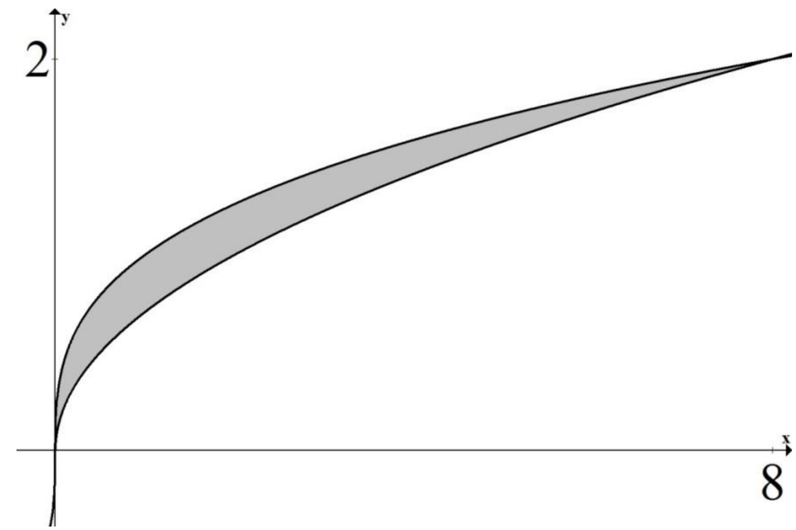


Using dy:



$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} (f(y_i) - g(y_i))\Delta y$$

Example: Set up an integral for the area bounded between $x = 2y^2$ and $x = y^3$ (shown below) using dy.



Summary: The area between curves

Step 1: Draw picture finding all intersections.

Step 2: Choose variable:

- same top/bot throughout
  →choose dx (and label *x*)
- same right/left throughout
  →Choose dy (and label *y*)

Get ***everything*** in terms of the variable you choose!!! And draw a typical approx. rectangle.

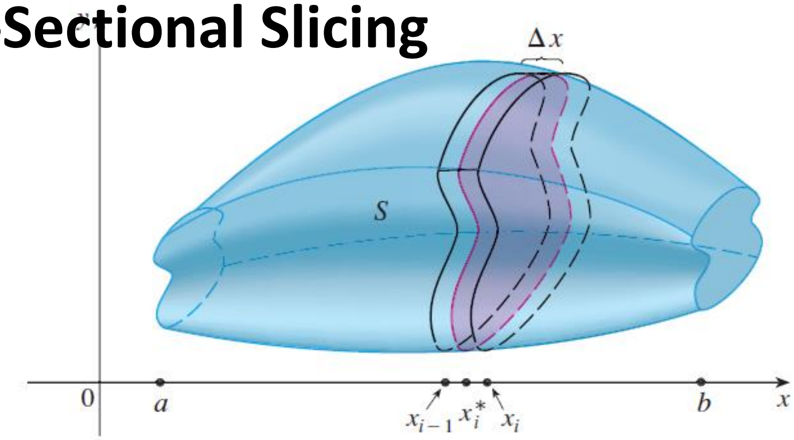
Step 3: Appropriately fill in

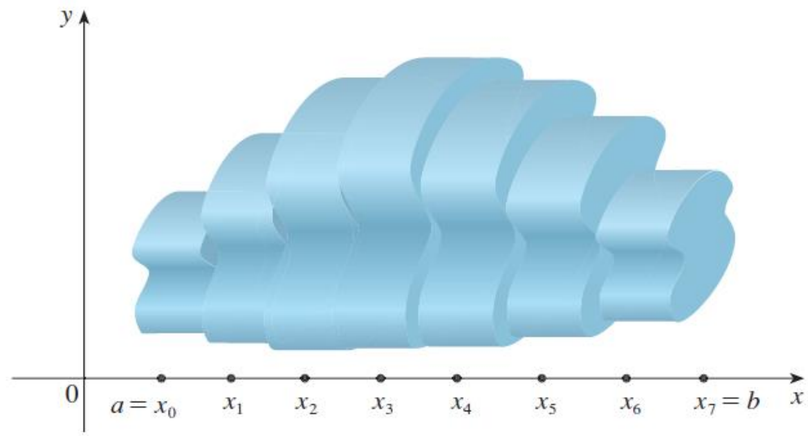
$$\text{Area} = \int_a^b (\text{TOP} - \text{BOTTOM})\, dx$$

$$\text{Area} = \int_c^d (\text{RIGHT} - \text{LEFT})\, dy$$

Example: Set up an integral (or integrals) that give the area of the region bounded by $x = y^2$ and $y = x - 2$.

6.2 Finding Volumes Using Cross-Sectional Slicing





If we can find the general formula, $A(x_i)$, for the area of a cross-sectional slice, then we can approximate volume by:

$$\text{Volume of one slice} \approx A(x_i)\,\Delta x$$

$$\text{Total Volume} \approx \sum_{i=1}^{n} A(x_i)\,\Delta x$$

This approximation gets better and better with more subdivisions, so

$$\text{Exact Volume} = \lim_{n\to\infty} \sum_{i=1}^{n} A(x_i)\,\Delta x$$

$$\text{Volume} = \int_a^b A(x)\,dx =$$

$$\int_a^b \text{"Cross-sectional area formula"}\,dx$$

Volume using cross-sectional slicing

1. Draw region and all intersections.
   Cut **perpendicular** to rotation axis.

   Label *x* if it cuts across the *x*-axis (and *y* if *y*-axis). Label **everything** in terms this variable.

2. Formula for cross-sectional area?

disc: Area = $\pi(\text{radius})^2$

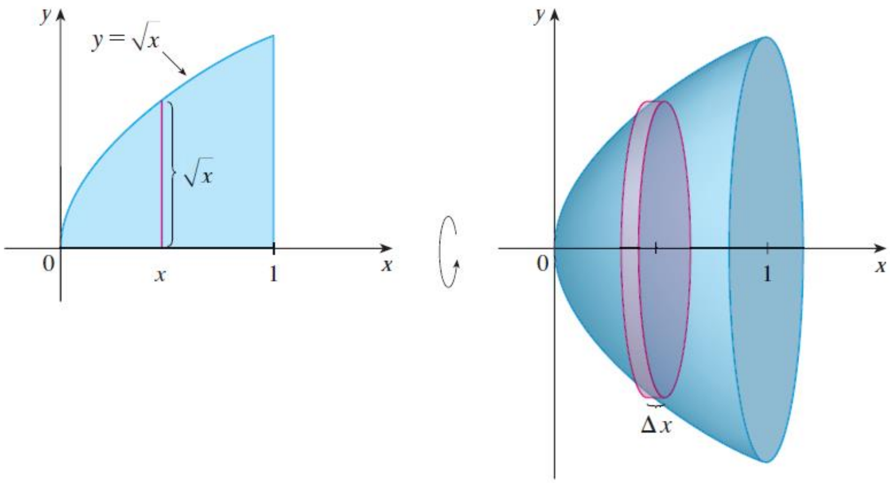
washer: Area = $\pi(\text{outer})^2 - \pi(\text{inner})^2$

square: Area = (Height)(Length)

triangle: Area = ½ (Height)(Length)

3. Integrate the area formula.

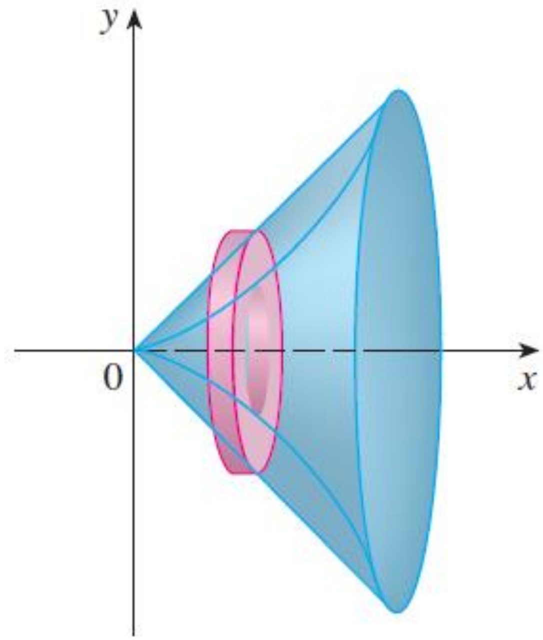
Example: Consider the region, *R*, bounded by $y = \sqrt{x}$, y = 0, and x = 1. Find the volume of the solid obtained by rotating R about the **x-axis**.



(a) (b)

Example: Consider the region, R, bounded by $y = \sqrt{x}$, y = 0, and x = 1. Find the volume of the solid obtained by rotating R about the ***y-axis***.

Example: Consider the region, R, bounded by $y = x$ and $y = x^4$. Find the volume of the solid obtained by rotating R about the **x-axis**.



Example: Consider the region, *R*, bounded by $y = x$ and $y = x^4$. (*R* is the same as the last example).

(a) Now rotate about the horizontal line y = -5. What changes?

(b) Now rotate about the horizontal line y = 10. What changes?

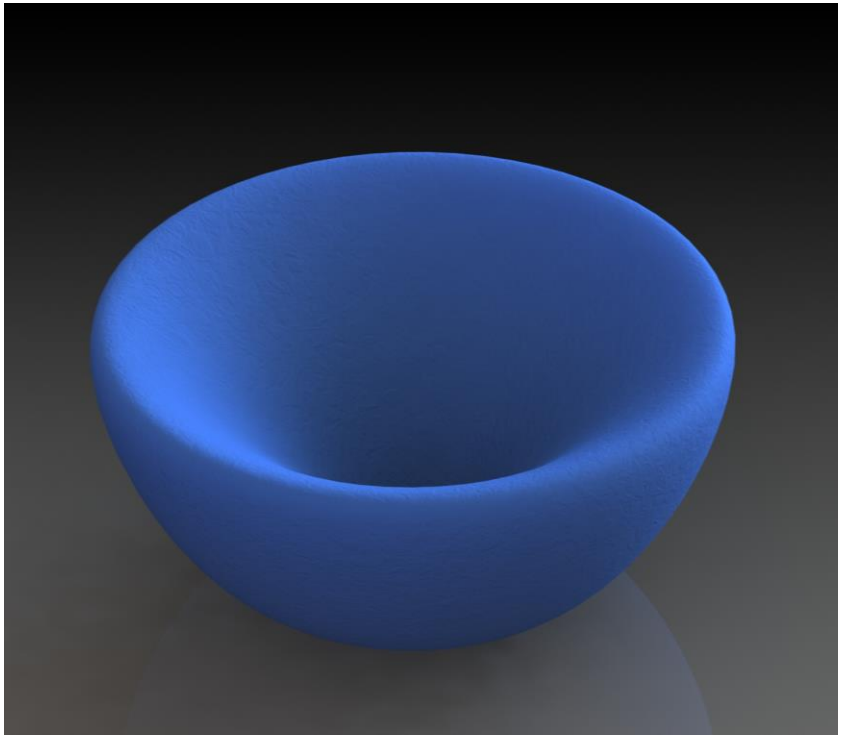
Example:

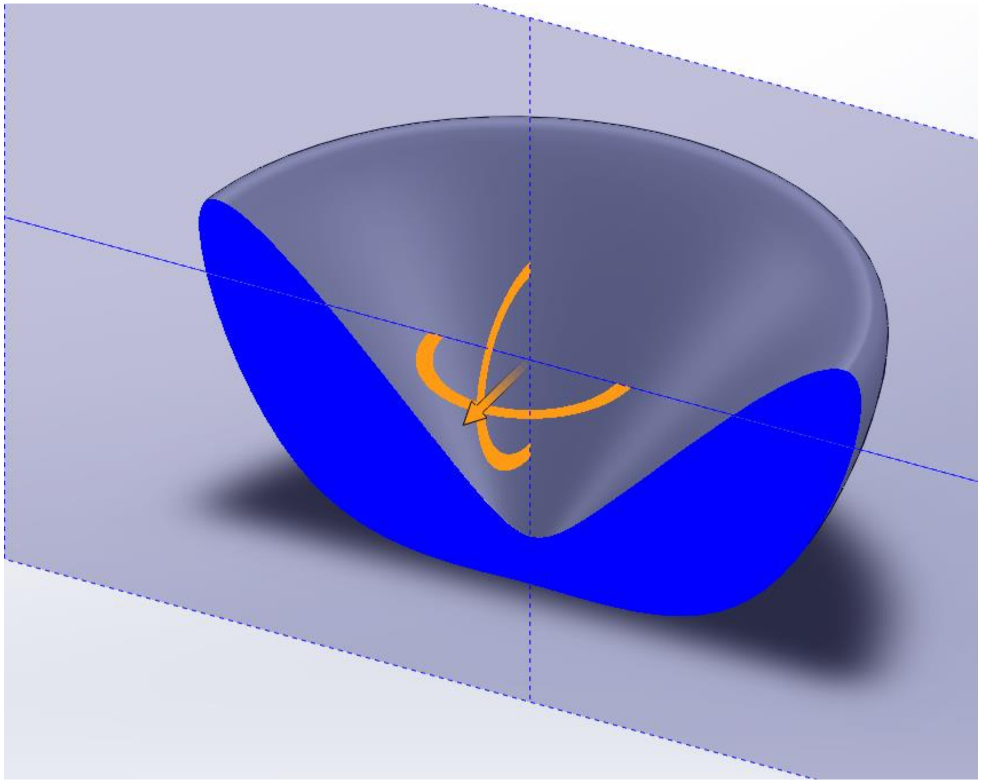
Consider the region bounded by

$$4x = y^2 \text{ and } y = 2x^3.$$

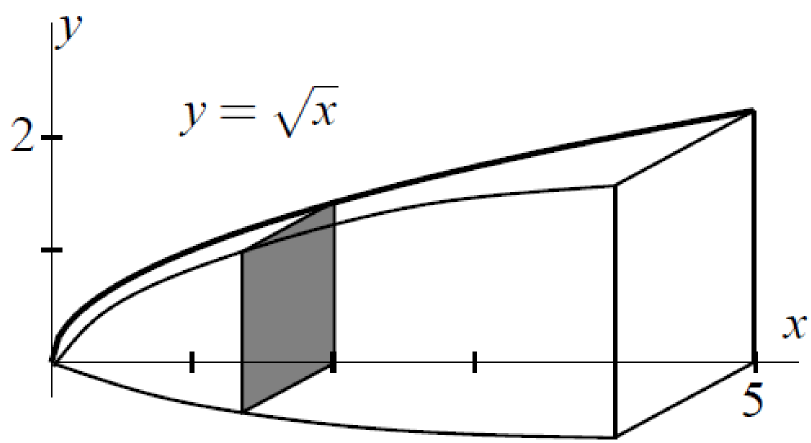
Find the volume of the solid obtained by rotating this region about the *y*-axis.

Visuals of the last example (created by one of my Math 125 student in the fall of 2017)





Example:
(From an old final and homework)
Find the volume of the solid shown.
The cross-sections are squares.



Summary (Cross-sectional slicing):

1. Draw Label
2. Cross-sectional area?
3. Integrate area.

This method has a major limitation:
6.2 method about *x-axis*, must use *dx*.
6.2 method about *y-axis*, must use *dy*.

What if the regions is rotated about the *x*-axis and we need to use *dy*?
(or about *y*-axis and we need dx?)
In these cases, 6.2 "Cross-sectional slicing" wouldn't work!

We need another method.
That is what we will do in 6.3.